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TECHNICAL NOTE

Bubble detachment criteria: some criticism of 'Das Abreissen von Dampfblasen an festen Heizflächen'

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Abstract—This note discusses criteria for bubble detachment in general and presents some criticism on the article 'Das Abreissen von Dampfblasen an festen Heizflächen' of J. Mitrovic [*Int. J. Heat Mass Transfer* **26**(7), 955–963 (1983)]. Force balances on a bubble as a whole should only be used in conjunction with some special bubble shape that only occurs at detachment if detachment criteria are to be derived. The criterion of Mitrovic is not useful.

1. INTRODUCTION

As boiling heat transfer has a manifold occurrence in various industrial processes, fundamental knowledge of bubble detachment is important. Mitrovic [1] considered the relatively simple system of a slowly growing, single bubble with its foot fixed in place without liquid flow and without the influence of other bubbles. It was attempted in this article, entitled 'Das Abreissen von Dampfblasen an festen Heizflächen', to set up the governing force balances and to devise a detachment criterion, irrespective of the orientation of heated plane wall with respect to gravity.

In this note this analysis and criterion are analysed. The governing equations are derived, both for the general case as for the thought experiment Mitrovic invented. Minor technical errors, such as a missing exponent $\frac{1}{2}$ in (30) and (33) and a missing term g in his (37) are not considered here.

The detachment criteria that have been found in the literature (see, for example, [2–6]) are studied and commented upon.

2. FORCES BALANCES AND BUBBLE DETACHMENT

2.1. *Introduction*

Among the forces that act on a bubble that has a footing at a plane wall are two that are usually [3] recognised as stemming from the integral of the static pressures over the bubble boundary that is only partially a liquid–gas interface. If the usual form of the buoyancy force, $V_g(\rho_l-\rho_g)g$ is retrieved from this integral a second force term results, the so-called buoyancy correction force. Here V denotes the system volume, g the magnitude of the acceleration due to gravity, ρ the mass density and the indices g and l denote the gas or vapour and the liquid, respectively. This procedure is made more clear in the following. The main difference of the equations of Mitrovic [1], henceforth denoted by M, with those of others [2, 3] seems to be the treatment of this buoyancy correction term and of the surface tension force, K_σ. The latter pulls the bubble towards the wall with component normal to the wall (only this component is used throughout this note if no vector-sign is used) given by:

$$K_\sigma \overset{\text{def}}{=} \int_{\text{foot}} \sigma \cdot \sin(\beta)\, \mathrm{d}s \tag{1}$$

where the integral is over the (1D) circumference of the foot of the bubble at the wall. Contact angle β is defined in the liquid, see Fig. 1. In this note, as far as possible the same symbols are used as in M.

The way M deals with K_σ and the buoyancy correction term is a bit obscured by the somewhat loose definitions of the important parameters K_0 and K_{l0} that he uses. These definitions are therefore first made more precise and the consequences analysed.

2.2. *Definitions*

Mitrovic wanted to show that at detachment the pressure in the liquid in the proximity of the bubble foot is equal to that of the gas–vapour at the foot. Or, to put it differently, that

$$\Delta K_0 = 0$$

is a useful detachment criterion, with ΔK_0 given by (again, only components normal to the wall are considered)

$$\Delta K_0 \overset{\text{def}}{=} \int_{\text{foot}} (p_g - p_l)\, \mathrm{d}A. \tag{2}$$

M does not use (2) to define ΔK_0. Just above his equation (27), henceforth denoted as (M27), he defines ΔK_0 as $K_0 - K_{l0}$. However, both K_0 and K_{l0} are not defined properly by M. If K_{l0} is defined by (3):

$$K_{l0} \overset{\text{def}}{=} \int_{\text{foot}} p_{l0}\, \mathrm{d}A \tag{3}$$

equation (M26) for a truncated sphere immediately follows:

$$\text{(M26)} \quad K_{l0} = \pi r^2 p_{l0} \sin^2 \beta. \tag{4}$$

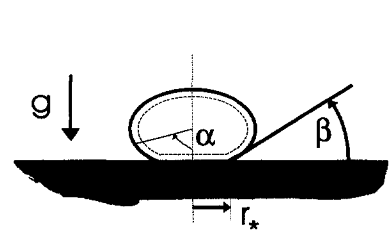

Fig. 1. Schematic of bubble on a horizontal wall and definition of the system boundary of [1].

NOMENCLATURE

a	acceleration
A	surface area
g	acceleration due to gravity
$\hat{\mathbf{g}}$	unit vector in the direction of gravity: $\vec{g}/\lvert\vec{g}\rvert$
K	force
m	mass
M	Mitrovic [1]
p	pressure
Δp	pressure drop
r	radius
r_*	radius of contact area, Fig. 1
R	radius of curvature
s	1D coordinate along circumference
t	time
u	velocity
V	volume
z	coordinate.

Greek symbols

β	contact angle, Fig. 1
η	dynamic viscosity
σ	surface tension
ρ	mass density.

Subscripts

1,2,3	corresponding to system 1, 2, 3, Figs. 2–4
cap	corresponding to the capillary at which the bubble is formed; at the bubble foot
d	dynamic
foot	bubble foot
g	gas–vapour content of bubble (is without subscript in [1])
l	liquid
l0	liquid in the close proximity of the bubble foot
n	in the normal direction
0	in the plane of the bubble foot
rope	corresponding to the rope
z	component in z-direction
σ	due to surface tension.

Here p_{l0} denotes the static liquid pressure at the wall just outside the bubble and β the static contact angle, see e.g. Fig. 1. Note that only bubbles are considered that are symmetrical around a vertical axis. Because of (3) there is no need for a derivation of (4) like the one M presents, starting from his equation (25).

So equation (3) seems to be a proper definition for K_{l0}. Trying to find one for K_0, let us put

$$K_0 \stackrel{\text{def}}{=} -\int_{\text{foot}} p_g \, d\vec{A} \cdot \hat{\mathbf{g}} \tag{5}$$

where the integral is over the bubble foot only and the minus sign is needed to make K_0 positive since the normal, $\vec{n}$, is taken inward; $d\vec{A} = \vec{n}\ dA$ and $\hat{\mathbf{g}} = \vec{g}/\lvert\vec{g}\rvert$. See Fig. 1 that corresponds to Fig. 4 of M, henceforth denoted as Fig. M4. By defining K_{l0} in this way (2) is correct and M's expression (24) for K_0 is easily derived, as shown below:

$$\text{(M24)} \quad K_0 = \int_{V_g} g\rho_g \, dV - \int_A \left(p_{l0} - \rho_l g z + 2\frac{\sigma}{r}\right) \cdot \cos\alpha \, dA. \tag{6}$$

In M and throughout this note it is assumed that the net acceleration of the bubble, $m_g\, a_g$, is negligible, i.e. that

$$\left(\frac{m_g + \rho_l V_g}{\rho_g V_g}\right)\lvert\vec{a}\rvert \ll \lvert\vec{g}\rvert. \tag{7}$$

Newton's second law

$$m_g \vec{a}_g = \sum_i \vec{K}_i \tag{8}$$

as applied to the system defined by the dotted line in Fig. 1, the bubble inside, with this assumption (7) reduces to

$$\int_{\text{foot}} p_g \, d\vec{A} \cdot \hat{\mathbf{g}} + \int_{g-f} p_g \, d\vec{A} \cdot \hat{\mathbf{g}} + \int_{V_g} g\rho_g \, dV = 0 \tag{9}$$

with g–f denoting the gas–fluid interface. The term $\cos\alpha$ in (6) $\equiv$ (M24) accounts for the normal if the bubble is a truncated sphere. In that case (9) yields (M24). In other cases $\cos\alpha$ doesn't intentionally account for the normal, but this loose definition in M has no consequences since other cases are not considered in M.

So equation (5) seems to be a proper definition for K_0. The combining of equations (3) and (5) yields (2) as the proper definition of ΔK_0. Since K_{l0} is an imaginary quantity, ΔK_0 at best can be regarded as the difference of the forces experienced at the area of the bubble foot just prior to and directly after bubble detachment. However, these two states of bubble growth and detachment have, *a priori*, little to tell about the process of detachment and the forces involved.

It is noted that the definition (5) of K_0 can not be described as 'die Differenz zwischen den senkrecht auf die Körperunterlage (bubble foot) wirkenden Volumen- und Oberflächenkräften' (M, page 958, left column, Section 3.2). The volume force acts on the center of mass and not on an area like the bubble foot† since for a stationary, non-deforming bubble with mass m_g

$$\int \rho_g \vec{g} \, dV = \int \rho_g \vec{a} \, dV = \frac{d^2}{dt^2} \int \rho_g \vec{r} \, dV = \frac{d^2}{dt^2} m_g \vec{R}_{CM}$$

with $\vec{R}_{CM} \stackrel{\text{def}}{=} \int \rho_g \vec{r} \, dV / m_g$ as the location of the center of mass and $\vec{r}$ the position vector of an arbitrary point in the bubble with acceleration $\vec{a}$.

It is also noted that if a bubble is growing, the system boundary defined in Fig. 1 is all but ideal. The thing that is observed to detach from a plate is the fluid boundary or, to put it differently, Gibb's dividing surface. If the system boundary is put in the liquid just outside the fluid–vapour interface, the attaching surface tension force appears explicitly in the governing equations. It is well known that each increase in interfacial boundary area corresponds to an increase in surface energy for which a force has to be applied. If the action of surface tension via the Laplace equation changes in time locally the gas pressure inside a bubble does so too. It would be difficult to find expressions for p in the system of Fig. 1 that describe this without resorting to the fluid phase again, that is without inclusion of the interface in the system.

2.3. *Analysis of the detachment criterion of Mitrovic*

Let the system boundary be taken outside the bubble and let $\vec{K}_d$ by definition represent the sum of all dynamic forces on

† This contradiction in terms also appears at other places in M.

the bubble including the added mass force. Note that $\vec{K}_d$ is zero for a free, spherical bubble for reasons of symmetry. Application of Gauss' theorem yields for the force components on the bubble in the direction of $\vec{g}$

$$\sum_i K_i = 2\pi r_* \sigma \sin\beta - \sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\pi r_*^2 - V_g(\rho_l - \rho_g)g - K_d. \quad (10)$$

Here V_g denotes the volume of the bubble and R_1 and R_2 are the principle radii of curvature at the bubble foot. The first term on the right-hand side (RHS) of equation (10) is K_σ, the second term the corrected buoyancy force. This term is needed to enable the application of the theorem of Gauss to the system boundary as if it were completely surrounded by liquid. The integral resulting from this theorem yields the $V_g\rho_l g$-term, since:

$$\int_{\text{gas-liquid interface}} p_l\,dA = \int_{\text{entire interface}} p_l\,dA - \int_{\text{foot}} p_l\,dA = V_g\rho_l g - \int_{\text{foot}} p_l\,dA.$$

With negligible bubble acceleration, see equations (7), (8) and (10) combine to yield for a truncated sphere with radius r

$$0 = 2\sigma\pi r \sin^2\beta - \sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\pi r_*^2 - \tfrac{4}{3}\pi g r^3(\rho_l - \rho_g)\cdot F(\beta) \quad (11)$$

where $F(\beta) \stackrel{\text{def}}{=} \frac{1}{2} + \frac{3}{4}\cos\beta - \frac{1}{4}\cos^3\beta$. The RHS of (11) would equal the RHS of (M27):

$$\text{(M27)} \quad \Delta K_0 = 2\sigma\pi r \sin^2\beta - \tfrac{4}{3}\pi g r^3(\rho_l - \rho_g)\cdot F(\beta)$$

if the corrected buoyancy term would be added to (M27). So the bubble detachment criterion of M, $\Delta K_0 = 0$ with ΔK_0 evaluated from (M27) is wrong even for his simplified case of a truncated sphere for two reasons:

(1) The corrected buoyancy term is missing. In slow growth bubble detachment from horizontal plane walls this term might be insignificant. In detachment from vertical plane walls it is not [3].

(2) The force balance (8) leading to (M27) should be satisfied *at all times* prior to detachment. It is just Newton's second law applied to an adhering bubble. Since such a force balance should be valid at all times it can not indicate bubble detachment.

Point 2 is missed by other authors as well (see, for example, [4]). It is further discussed in Section 2.5.

The use of $\Delta K_0 = 0$ as a detachment criterion leads to erroneous results. Equation (M13) for a bubble like the one of Fig. 1 is an example. It is stated without a derivation and reads

$$\text{(M13)} \quad \Delta K_0 = 2\pi r_* \sigma \sin\beta - V_g(\rho_l - \rho_g)g - K_d + \Delta p_d \pi r_*^2. \quad (12)$$

The dynamic pressure drop, Δp_d, is part of K_d and unimportant here since it will be neglected anyway. With the aid of (10) (M13) is easily cast into the form

$$\Delta K_0 = \sum_i K_i + \sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\pi r_*^2 + \Delta p_d \pi r_*^2 = \sum_i K_i + \pi r_*^2\left(\Delta p_d + \sigma\left\{\frac{1}{R_1} + \frac{1}{R_2}\right\}\right).$$

In the case of slow bubble growth, see equation (7), the terms $\Sigma_i K_i$ and Δp_d are negligible compared to gravity and surface tension. M's criterion thus leads to

$$\sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = 0$$

which can never be satisfied.

Another example is the 'Gedanken', thought, experiment of M discussed in the next section.

2.4. *The Gedanken experiment of Mitrovic*

Mitrovic made some simplifications in his analysis of the example of Fig. M6, see Fig. 2. The most important one is the neglection of the surface tension force defined by equation (1). In the following, this example is studied with the force balances of three systems in order to be able to value the importance of K_σ and of the buoyancy correction term.

The total force on system 1 as defined by the dotted line in Fig. 2 is given by Newton's second law as

$$m\vec{a} = \sum_i \vec{K}_i = \int p_l\,d\vec{A} + \vec{K}_{\text{rope}} + m\vec{g}. \quad (13)$$

The integral is over the system boundary excluding the rope with the normal taken inward in the system, as before. In (13) m denotes the total mass of system 1 and $\vec{K}_{\text{rope}}$ represents the force on the system by the rope. Pressure, p, comprises static and dynamic pressures (including added mass). The dynamic pressures are neglected since only hardly accelerated systems are considered to satisfy equation (7). Let z denote the vertical direction against gravity, normal to the plate (see Fig. 2). Application of the theorem of Gauss yields for the components in z-direction:

$$m a_z = V_g(\rho_l - \rho_g)g + \tilde{K}_{\text{rope}} + V_3(\rho_l - \rho_3)g. \quad (14)$$

Here index 3 refers to the plate and

$$\tilde{K}_{\text{rope}} \stackrel{\text{def}}{=} K_{\text{rope}_z} + p_l \cdot A_{\text{rope}}$$

with A_{rope} the cross-sectional area of the rope at the plate where p_l has to be evaluated. The A_{rope}-term in $\tilde{K}_{\text{rope}}$ is due to the extending of the integral in equation (13) to the entire system boundary. The value of $\tilde{K}_{\text{rope}}$ is selected in case (a) of M such that it balances the weight and the buoyancy of the plate, $V_3(\rho_l - \rho_3)g$ (named K by M). The net force in M's case (a) (named ΔK by M) is therefore given by $V_g(\rho_l - \rho_g)g$ as is correctly stated in (M35):

$$\text{(M35)} \quad \Delta K = V_g(\rho_l - \rho_g)g.$$

In M ρ denotes ρ_g and V_g is written in terms of an (equivalent) radius.

Now consider system 2, the bubble, as defined by the dotted boundary in Fig. 3. Let $K_{\sigma z}$ denote the z-component

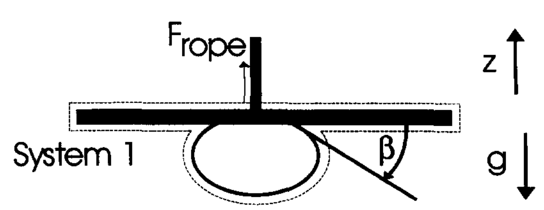

Fig. 2. Schematic of thought experiment and definition of system 1.

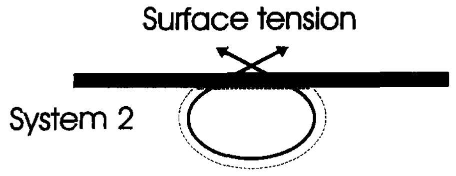

Fig. 3. Definition of system 2 in the experiment of Fig. 2.

of the adhering surface tension force at the bubble foot. System 2 experiences the following net force in z-direction:

$$K_{\sigma z}+V_g g(\rho_l-\rho_g)-(p_g-p_l)A_{cap} \qquad (15)$$

where the minus sign of the last term is due to the inner product of the unit vector in z-direction with the normal at the bubble foot; A_{cap} denotes the area of the bubble foot. The last term is the corrected buoyancy term needed to enable the application of the theorem of Gauss to the system boundary as if it were completely surrounded by liquid. The resulting integral yields the $V_g g \rho_l$-term as before. The pressures in the corrected buoyancy term have to be evaluated at A_{cap}.

A similar procedure for system 3 as defined by the dotted line in Fig. 4, the plate, yields the following net force component:

$$\vec{K}_{rope}-K_{\sigma z}+V_3 g(\rho_l-\rho_3)+(p_g-p_l)A_{cap}. \qquad (16)$$

Once again the last term is a buoyancy correction term, here imaginarily to fill up the boundary of system 3 with liquid. Note that the sum of the forces components on systems 2 and 3 equals the force on system 1, as it should.

Although the system boundary drawn in Fig. M6 [for both cases (a) and (b) the same] suggests differently, M merely considers forces on the plate, so merely considers system 3 as follows from sentences as '... wirkt die Kugel auf die Platte...', found just above (M35), p. 960. He probably assumes that the Archimedes force acting on the center of mass of the bubble can be transferred to the point where the bubble touches the plate. A real bubble, however, is not stiff, making such a transfer impossible. The action of the external force, gravity, is on the center of mass as shown above in Section 2.2. So M seems to analyse system 3 but omits the surface tension force since after using $\vec{K}_{rope}+V_3 g(\rho_l-\rho_3)=0$ for the net force here given by (16) he comes up with (M36):

$$\text{(M36)} \quad \Delta K=(p_g-p_l)A_{cap}. \qquad (17)$$

This is wrong since $K_{\sigma z}$ is missing.

M merely computes forces for a truncated sphere with radius r. In this specific case

$$A_{cap}(p_g-p_l)-K_\sigma \approx A_{cap}\frac{2\sigma}{r}-2\pi r\sigma \sin^2(\beta)=0 \qquad (18)$$

since $A_{cap}=\pi\{r\sin(\beta)\}^2$. Note that p_g exceeds p_l at the bubble foot. So in this case the second term balances the fourth in (16) and no net force is experienced by the plate at all. The magnitude of the net force on the total system is that of the net force on the bubble in this case. Only if the bubble boundary would be stiff this would lead to systems 2 and 3 having the same acceleration. In reality, the bubble interface is not stiff and only the center of mass of the bubble is a little accelerated. The bubble assumes a different shape that allows the plate to be a little accelerated as well. If $\vec{K}_{rope}$ is taken to be zero, the big difference in mass of systems 2 and 3 causes the bubble to become flattened and to assume a shape in which $\vec{K}_\sigma \cdot \vec{g}=0$, i.e. $K_{\sigma z}=0$ (see Fig. 5 that also defines the bubble height, h). The net force on the plate is in that case

$$(p_g-p_l)\cdot A_{cap} \simeq (\rho_l-\rho_g)\cdot gh\pi r_*^2$$

since the bubble is flat at the bottom, making $p_g \simeq p_l+\rho_l gh$ there. This force on the plate is produced by the bubble since (15) yields

$$0=V_2 g(\rho_l-\rho_g)-(p_g-p_l)A_{cap}$$

if the bubble center of mass does not move. If the bubble is flattened and the weight of the plate is compensated by $\vec{K}_{rope}$, like in M, the plate is accelerated by the net force $(\rho_l-\rho_g)\cdot gh\pi r_*^2$.

That the truncated sphere is an exceptional case is also seen if the plate is placed vertically. Because of the axial symmetry of the truncated sphere $\vec{g}\cdot\vec{K}_\sigma=0$ in this case and no adhering force exists. Any bubble with an axisymmetrical shape would therefore escape immediately from the vertical wall. Nature solves this anomaly by not allowing for such a symmetry (see Fig. 6).

2.5. *Other detachment criteria*

One detachment criterion has been found in the literature that is not based on the force balance of an attached bubble as a whole. It was derived by Chesters [2, 5]. In this approach, bubble shapes are computed for bubbles generated at a sharp-edged capillary mouth from the normal pressure balance at the interface and the static liquid pressure assuming some radius of curvature at the top of the bubble. The computed interface is convex near the top and changes to a concave shape at, roughly speaking, what could be the bubble foot. The radius of the bubble foot, r_*, is prescribed and equals the radius of a given capillary. A family of computed interfaces, all with different radii of curvature at the top, is matched to r_*. If no shape fits bubble detachment is said to occur.

This recipe to predict the bubble shape and volume at detachment requires the evaluation of many computations of the same kind. It is not a single, straightforward compu-

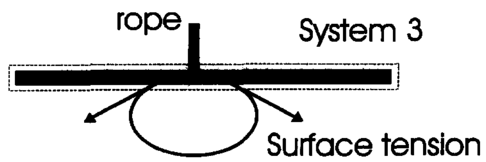

Fig. 4. Definition of system 3 in the experiment of Fig. 2.

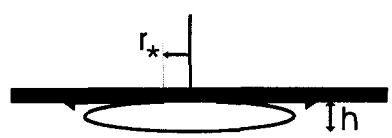

Fig. 5. Schematic plan-view of actual bubble flattening.

Fig. 6. Schematic of bubble adhering on a cavity in a vertical wall.

tation. It can therefore be described as an *a posteriori* criterion.

All other detachment criteria seem to be based on force balances of an attached bubble as a whole. They therefore rely on the accuracy, or rather the inaccuracy, with which the individual force terms are known just prior to detachment since Newton's second law on which the force balances are based should be satisfied *at all times* prior to detachment. This fact is utilised by van Helden *et al.* [3] to enlarge the accuracy with which the individual force components are known.

It would be possible to devise an *a priori* detachment criterion based on both the geometry of a bubble and a force balance for the bubble as a whole if a certain bubble shape could be recognised as occurring *only* at detachment. See, for example, Fig. 7. Suppose that the shape with the neck perpendicular to the cavity mouth only occurs at detachment, when liquid is flowing towards this neck. Contact angle, β, is not static in this situation but dynamic and equals $\pi/2$. For slow growth, the overall force balance analoguous to (11) is easily derived as:

$$0 = 2\pi r_* \sigma - (p_{g0} - p_{l0})\pi r_*^2 - \tfrac{4}{3}\pi g r^3 (\rho_l - \rho_g) \tag{19}$$

where r represents the volume equivalent radius corresponding to the bubble volume above the neck. Since p_l is dynamic the normal stress balance has to be used to evaluate the corrected buoyancy term:

$$p_g = p_l - \sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - 2\eta(\vec{n}\cdot\vec{\nabla})\vec{n}\cdot\vec{u}_l$$

with the normal, $\vec{n}$, taken inward into the bubble, see Fig. 7. Let $\partial/\partial n$ denote $\vec{n}\cdot\vec{\nabla}$. For the special shape selected, $p_g = p_l + \sigma/r_* - 2\eta(\partial/\partial n)u_n$ with $(\partial/\partial n)\,u_n$ positive since close to the bubble the velocity is higher than that further away.

As a first approximation it is possible to assume that the terms σ/r_* and $2\eta(\partial/\partial n)u_n$ are compensating each other, which would yield $p_g \approx p_l$. A better estimate would only be possible if the physics leading to this special geometry of the attached bubble would be better understood. With the present approximations

$$r \simeq \sqrt[3]{\left(\tfrac{3}{2} r_* \frac{\sigma}{g(\rho_l - \rho_g)}\right)}. \tag{20}$$

It was experimentally determined by Blanchard and Syzdek [6] that (20) holds very well for air injection for small orifice radii, i.e. $r_* < 1.5$ mm, provided the bubble frequency is below 30 bubbles per minute. This indicates that the neck shape as shown in Fig. 7 might indeed be occurring *only* at detachment in some circumstances. It is, however, not intended here to prove this. The above derivation merely exemplifies the way a force balance, if applied to a special geometry, can be used to indicate and predict detachment conditions.

Note that consequence $p_g - p_l \approx 0$ in this case resembles the detachment criterion of Mitrovic [1]. However, the detachment criterion in the above example is the selection of a specific bubble shape that only occurs at detachment. The allegation that the pressure jump at the interface of the bubble neck is approximately zero is derived along different lines of reasoning.

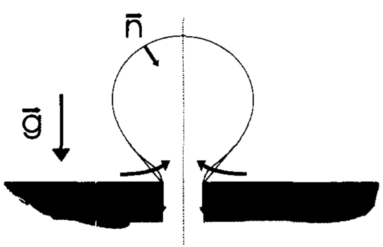

Fig. 7. Schematic of bubble detaching from a cavity in a horizontal wall.

3. CONCLUSIONS

The analysis of Mitrovic [1] is incorrect because of the neglect of the so-called corrected buoyancy force and an improper accounting for the surface tension force. His detachment criterion is not useful.

Most detachment criteria in the literature seem to be based on a force balance of a growing bubble as a whole. However, an appropriate force balance should be satisfied at all times prior to detachment. For this reason, a global force balance can only be used to devise a detachment criterion if a certain bubble shape can be found that occurs at detachment only.

The criterion of Chesters [2, 5] at present seems to mimic the physics of bubble detachment best.

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